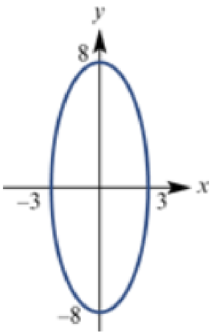
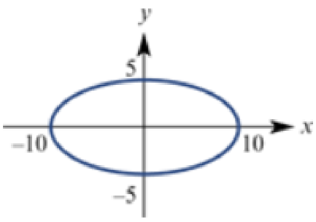
Exercise 12E: Solutions

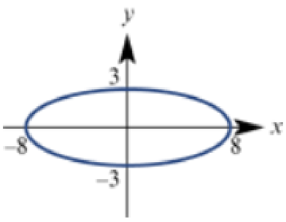
1 a



b

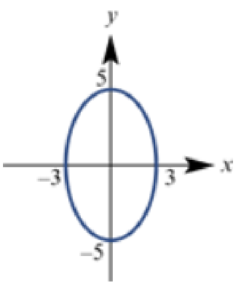


c

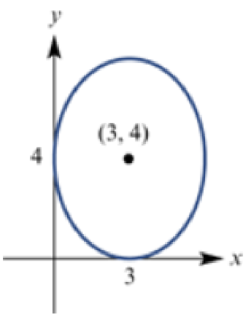


d Dividing both sides of the expression by 225 gives

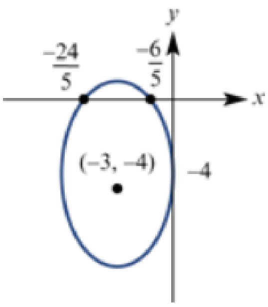
$$\frac{x^2}{9} + \frac{y^2}{25} = 1.$$



2 a



b



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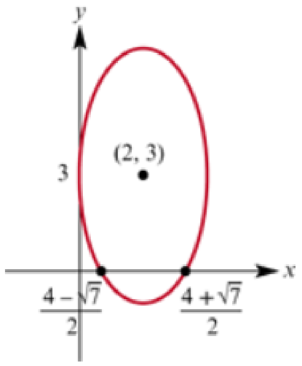
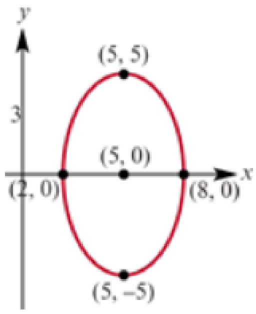


d



3 a The equation can be found by noting that the x-intercepts are $x = \pm 5$ and the y-intercepts are $y = \pm 4$. Therefore, the equation must be

$$\frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$$

$$\text{or } \frac{x^2}{25} + \frac{y^2}{16} = 1.$$

b The centre of the ellipse is $(2, 0)$. Therefore, the equation of the ellipse must be

$$\frac{(x-2)^2}{3^2} + \frac{y^2}{2^2} = 1$$

$$\text{or } \frac{(x-2)^2}{9} + \frac{y^2}{4} = 1.$$

c The centre of the ellipse is $(-1, 1)$. Therefore, the equation of the ellipse must be

$$\frac{(x+1)^2}{2^2} + \frac{(y-1)^2}{1^2} = 1$$

$$\text{or } \frac{(x+1)^2}{4} + (y-1)^2 = 1.$$

4 Let (x, y) be the coordinates of point P. If $AP + BP = 4$ then,

$$\sqrt{(x-1)^2 + y^2} + \sqrt{(x+1)^2 + y^2} = 4,$$

$$\text{then} \sqrt{(x-1)^2 + y^2} = 4 - \sqrt{(x+1)^2 + y^2}.$$

Squaring both sides gives,

$$(x-1)^2 + y^2 = 16 - 8\sqrt{(x+1)^2 + y^2} + (x+1)^2 + y^2.$$

Now expand and simplify to obtain

$$x^2 - 2x + 1 + y^2 = 16 - 8\sqrt{(x+1)^2 + y^2} + x^2 + 2x + 1 + y^2$$

$$-2x = 16 - 8\sqrt{(x+1)^2 + y^2} + 2x,$$

$$4x + 16 = 8\sqrt{(x+1)^2 + y^2}$$

$$x + 4 = 2\sqrt{(x+1)^2 + y^2}$$

Squaring both sides again gives

$$x^2+8x+16=4(x^2+2x+1+y^2).$$

Simplifying yields

$$12=3x^2+4y^2 \quad \text{or} \quad \frac{x^2}{4}+\frac{y^2}{3}=1.$$

This is an ellipse with centre the origin, and intercepts at $x=\pm 2$ and $y=\pm\sqrt{3}$.

5 Let (x,y) be the coordinates of point P. If $AP+BP=6$ then,

$$\sqrt{x^2+(y-2)^2}+\sqrt{x^2+(y+2)^2}=6,$$

$$\text{then}\sqrt{x^2+(y-2)^2}=6-\sqrt{x^2+(y+2)^2}$$

Squaring both sides gives,

$$x^2+(y-2)^2=36-12\sqrt{x^2+(y+2)^2}+x^2+(y+2)^2.$$

Now expand and simplify to obtain

$$\begin{aligned} x^2+y^2-4y+4 & \\ =36-12\sqrt{x^2+(y+2)^2}+x^2+y^2+4y+4 & \\ -4y=36-12\sqrt{x^2+(y+2)^2}+4y & \\ 8y+36=12\sqrt{x^2+(y+2)^2} & \\ 2y+9=3\sqrt{x^2+(y+2)^2} & \end{aligned}$$

Squaring both sides again gives

$$4y^2+36y+81=9(x^2+y^2+4y+4).$$

Simplifying yields

$$9x^2+5y^2=45 \quad \text{or} \quad \frac{x^2}{5}+\frac{y^2}{9}=1.$$

This is an ellipse with centre the origin, and intercepts at $x=\pm\sqrt{5}$ and y-intercepts $y=\pm 3$.

6 Let (x,y) be the coordinates of point P. If $FP=\frac{1}{2}MP$ then

$$\sqrt{(x-2)^2+y^2}=\frac{1}{2}\sqrt{(x+4)^2}.$$

Squaring both sides gives

$$\begin{aligned} (x-2)^2+y^2 &= \frac{1}{4}(x+4)^2 \\ 4(x^2-4x+4)+4y^2 &= x^2+8x+16 \\ 4x^2-16x+16+4y^2 &= x^2+8x+16 \\ 3x^2-24x+4y^2 &= 0. \end{aligned}$$

Completing the square gives,

$$\begin{aligned} 3(x^2-8x)+4y^2 &= 0 \\ 3((x^2-8x+16)-16)+4y^2 &= 0 \\ 3((x-4)^2-16)+4y^2 &= 0 \\ 3(x-4)^2+4y^2 &= 48 \end{aligned}$$

Or equivalently,

$$\frac{(x-4)^2}{16}+\frac{y^2}{12}=1.$$

7 The transformation is defined by the rule $(x, y) \to (5x, 3y)$. Therefore let $x' = 5x$ and $y' = 3y$ where (x', y') is the image of (x, y) under the transformation. Hence $x = \frac{x'}{5}$ and $x = \frac{y'}{3}$. The equation

$$x^2 + y^2 = 1$$

becomes,

$$\frac{(x')^2}{25} + \frac{(y')^2}{9} = 1$$

Ignoring the apostrophes gives,

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

This is an ellipse with centre the origin, with intercepts at $(\pm 5, 0)$ and $(0, \pm 3)$.